6.5 Prove Triangles Similar by SSS and SAS

Before You used the AA Similarity Postulate to prove triangles similar.

Now You will use the SSS and SAS Similarity Theorems.

Why? So you can show that triangles are similar, as in Ex. 28.

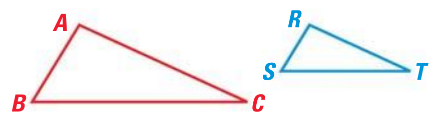
Key Vocabulary
- **ratio,** *p. 356*
- **proportion,** *p. 358*
- **similar polygons,** *p. 372*

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

THEOREM *For Your Notebook*

THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

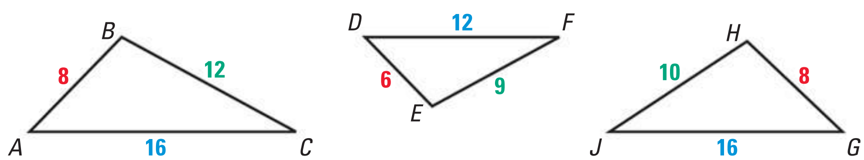


If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

Proof: p. 389

EXAMPLE 1 Use the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?



Solution

APPLY THEOREMS
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides	Longest sides	Remaining sides
$\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$	$\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$	$\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

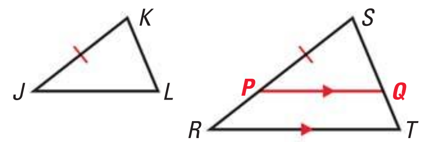
Shortest sides	Longest sides	Remaining sides
$\frac{AB}{GH} = \frac{8}{8} = 1$	$\frac{CA}{JG} = \frac{16}{16} = 1$	$\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

GIVEN ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

PROVE ▶ $\triangle RST \sim \triangle JKL$



USE AN AUXILIARY LINE

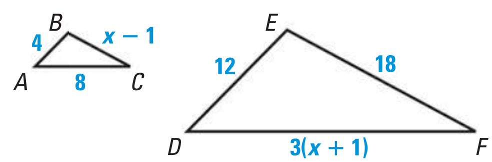
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Use the SSS Similarity Theorem

ALGEBRA **Find the value of x that makes $\triangle ABC \sim \triangle DEF$.**



Solution

STEP 1 **Find** the value of x that makes corresponding side lengths proportional.

CHOOSE A METHOD

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

$\frac{4}{12} = \frac{x-1}{18}$ — **Write proportion.**

$4 \cdot 18 = 12(x-1)$ — **Cross Products Property**

$72 = 12x - 12$ — **Simplify.**

$7 = x$ — **Solve for *x*.**

STEP 2 **Check** that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

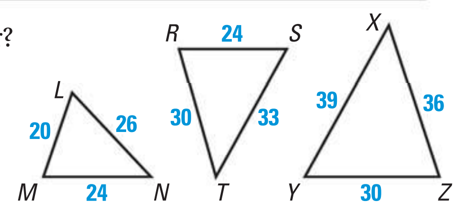
$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Which of the three triangles are similar? Write a similarity statement.

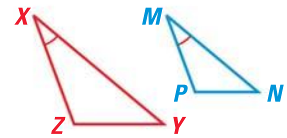
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.



THEOREM *For Your Notebook*

THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

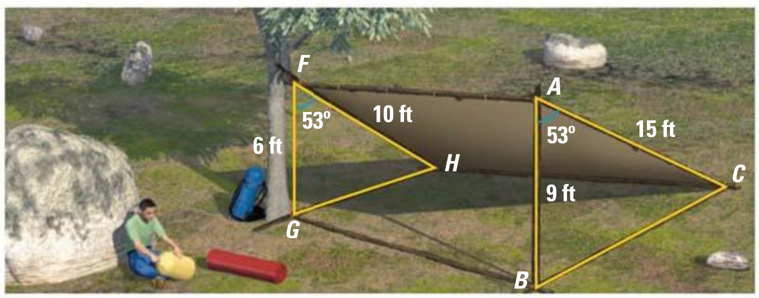


If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof: Ex. 37, p. 395

EXAMPLE 3 Use the SAS Similarity Theorem

LEAN-TO SHELTER You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



Solution

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$ **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

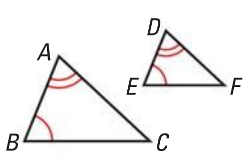
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▶ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

CONCEPT SUMMARY *For Your Notebook*

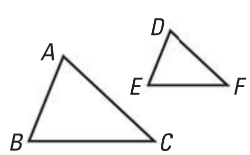
Triangle Similarity Postulate and Theorems

AA Similarity Postulate



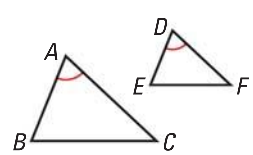
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem

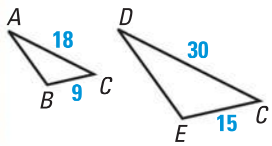


If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

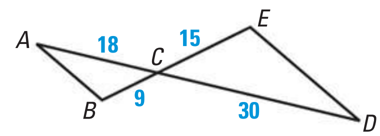
EXAMPLE 4 Choose a method

VISUAL REASONNING

To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.



Tell what method you would use to show that the triangles are similar.



Solution

Find the ratios of the lengths of the corresponding sides.

Shorter sides $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

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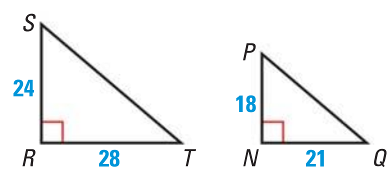
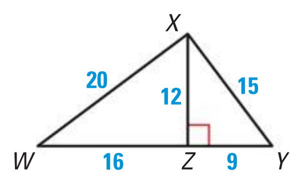
✓ GUIDED PRACTICE for Examples 3 and 4

Explain **how to show that the indicated triangles are similar.**

3. $\triangle SRT \sim \triangle PNQ$



4. $\triangle XZW \sim \triangle YZX$



6.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 14, 32, 34, and 36

SKILL PRACTICE

1. **VOCABULARY** You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.

2. ★ **WRITING** If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

EXAMPLES 1 and 2

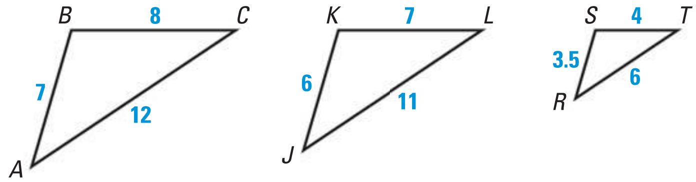
on pp. 388–389 for Exs. 3–6

SSS SIMILARITY THEOREM **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

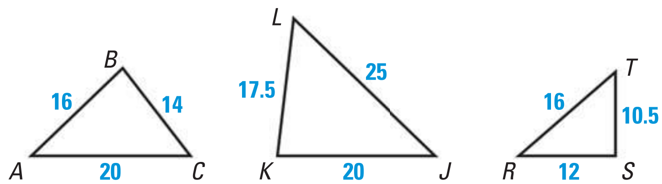
3. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

4. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

5. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



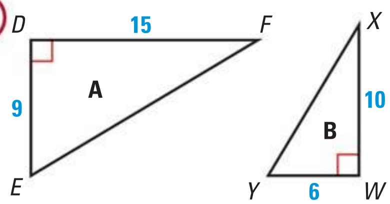
6. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



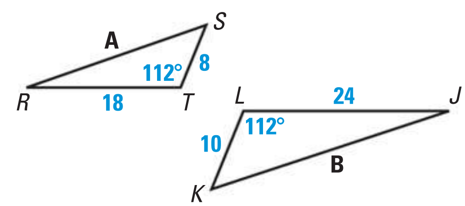
EXAMPLE 3
on p. 390
for Exs. 7–9

SAS SIMILARITY THEOREM **Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.**

7.



8.

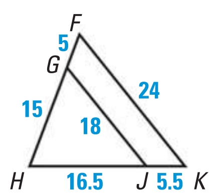


9. **ALGEBRA** Find the value of n that makes $\triangle PQR \sim \triangle XYZ$ when $PQ = 4$, $QR = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle Q \cong \angle Y$. Include a sketch.

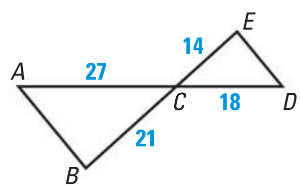
EXAMPLE 4
on p. 391
for Exs. 10–12

SHOWING SIMILARITY **Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.**

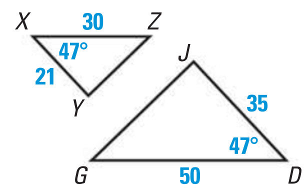
10.



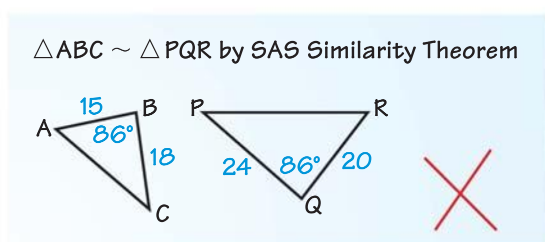
11.



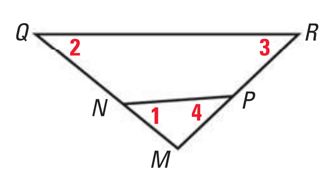
12.



13. **ERROR ANALYSIS** *Describe* and correct the student's error in writing the similarity statement.



14. ★ **MULTIPLE CHOICE** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true?



(A) $\angle 1 \cong \angle 2$ (B) $\overline{QR} \parallel \overline{NP}$

(C) $\angle 1 \cong \angle 4$ (D) $\triangle MNP \sim \triangle MRQ$

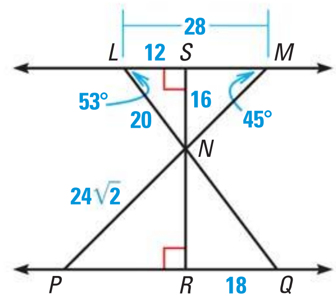
DRAWING TRIANGLES **Sketch the triangles using the given description. *Explain* whether the two triangles can be similar.**

15. In $\triangle XYZ$, $m\angle X = 66°$ and $m\angle Y = 34°$. In $\triangle LMN$, $m\angle M = 34°$ and $m\angle N = 80°$.

16. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

17. The side lengths of $\triangle ABC$ are 24, $8x$, and 54, and the side lengths of $\triangle DEF$ are 15, 25, and $7x$.

FINDING MEASURES **In Exercises 18–23, use the diagram to copy and complete the statements.**



18. $m\angle NQP = \underline{\ ?\ }$

19. $m\angle QPN = \underline{\ ?\ }$

20. $m\angle PNQ = \underline{\ ?\ }$

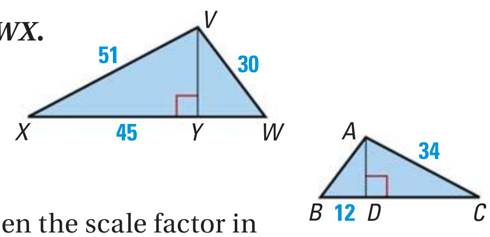
21. $RN = \underline{\ ?\ }$

22. $PQ = \underline{\ ?\ }$

23. $NM = \underline{\ ?\ }$

24. **SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar.

CHALLENGE **In the figure at the right, $\triangle ABC \sim \triangle VWX$.**



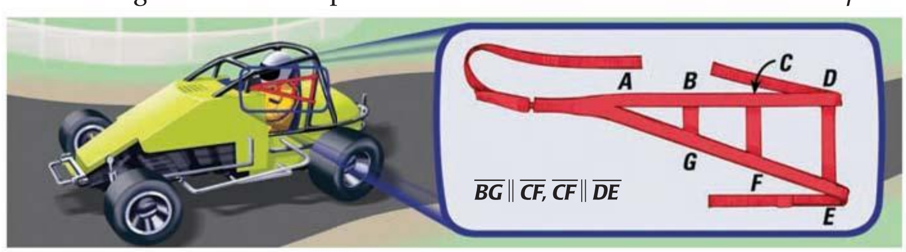
25. Find the scale factor of $\triangle VWX$ to $\triangle ABC$.

26. Find the ratio of the area of $\triangle VWX$ to the area of $\triangle ABC$.

27. Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. *Justify* your conjecture.

PROBLEM SOLVING

28. **RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? *Explain*.



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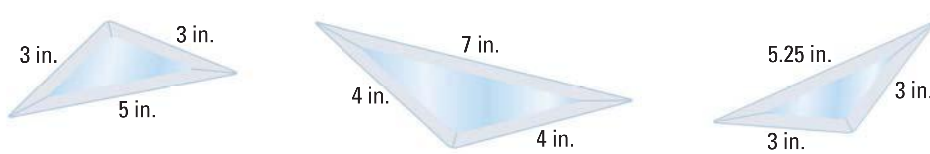
EXAMPLE 1 on p. 388 for Ex. 29

29. **STAINED GLASS** Certain sections of stained glass are sold in triangular *beveled* pieces. Which of the three beveled pieces, if any, are similar?



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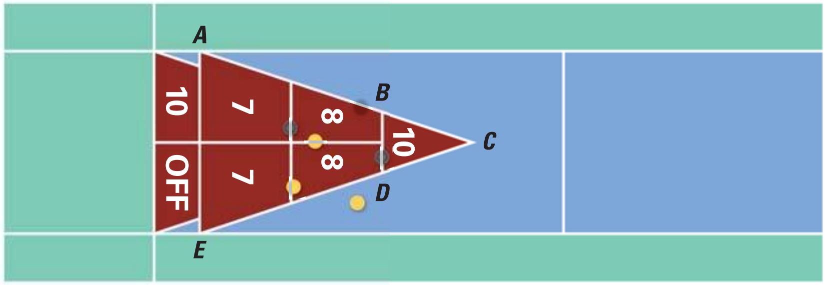
SHUFFLEBOARD In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.



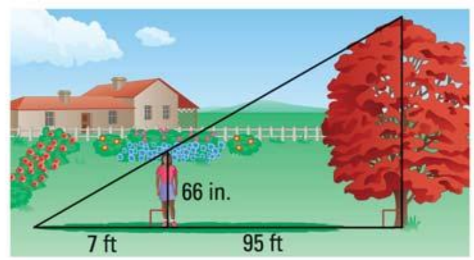
30. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem?

31. What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem?

32. ★ **OPEN-ENDED MATH** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate.

EXAMPLE 4
on p. 391
for Ex. 33

33. **MULTI-STEP PROBLEM** Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.



a. What postulate or theorem can you use to show that the triangles in the diagram are similar?

b. About how tall is the tree, to the nearest foot?

c. **What If?** Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree?

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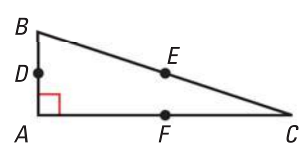
34. ★ **EXTENDED RESPONSE** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.

a. The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram.

b. Write the ratio of the lengths of the second pair of corresponding legs.

c. Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles?

35. **PROOF** Given that $\triangle ABC$ is a right triangle and D, E, and F are midpoints, prove that $m\angle DEF = 90°$.

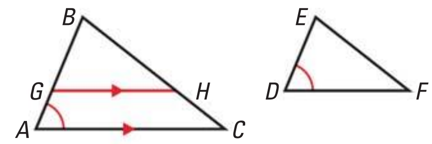


36. ★ **WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain.*

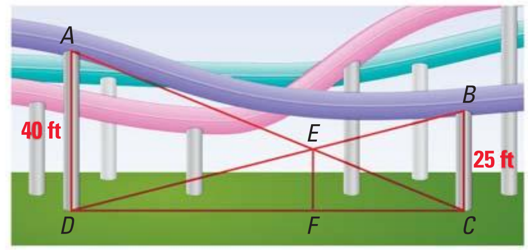
37. PROVING THEOREM 6.3 Write a paragraph proof of the SAS Similarity Theorem.

GIVEN ▶ $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$

PROVE ▶ $\triangle ABC \sim \triangle DEF$



38. CHALLENGE A portion of a water slide in an amusement park is shown. Find the length of $\overline{EF}$. (*Note:* The posts form right angles with the ground.)

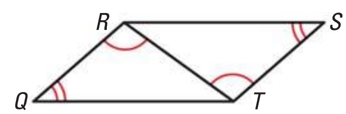


MIXED REVIEW

Find the slope of the line that passes through the given points. *(p. 171)*

39. (0, −8), (4, 16) **40.** (−2, −9), (1, −3) **41.** (−3, 9), (7, 2)

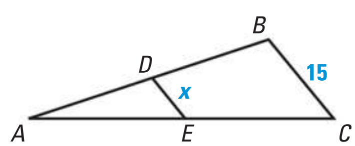
42. State the postulate or theorem you would use to prove the triangles congruent. Then write a congruence statement. *(p. 249)*



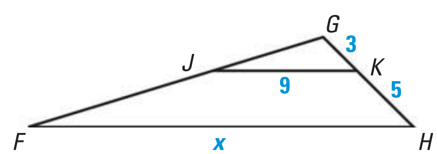
PREVIEW
Prepare for Lesson 6.6 in Exs. 43–44.

Find the value of *x*.

43. $\overline{DE}$ is a midsegment of $\triangle ABC$. *(p. 295)*



44. $\frac{GK}{GH} = \frac{JK}{FH}$ *(p. 364)*



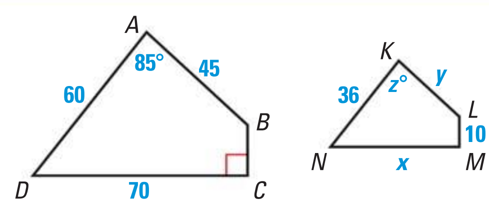
QUIZ *for Lessons 6.3–6.5*

In the diagram, *ABCD* ~ *KLMN*. *(p. 372)*

1. Find the scale factor of *ABCD* to *KLMN*.

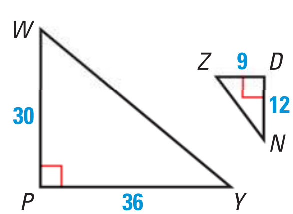
2. Find the values of *x*, *y*, and *z*.

3. Find the perimeter of each polygon.



Determine whether the triangles are similar. If they are similar, write a similarity statement. *(pp. 381, 388)*

4.



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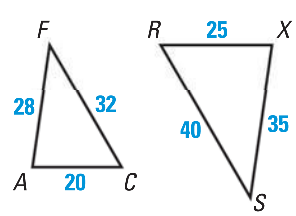

6.